Math 152, Fall 2022
Jo Hardin
WU # 5
Thursday 9/15/22

Your name: ____

Names of people you worked with: ____

Task:

Let $X_1, X_2, \ldots, X_n$ be distributed identically and independently $\exp(\theta)$. Find the value of θ that maximizes their joint distribution (i.e., the likelihood function).

$$f(x|\theta) = \theta e^{-x\theta} \qquad x > 0$$

Solution:

$$\begin{aligned}
f(x|\theta) &= \theta e^{-x\theta} \qquad x > 0 \\
f(\underline{x}|\theta) &= \theta^n e^{-\sum x_i \theta} \qquad \forall x_i > 0 \\
\frac{\partial f(\underline{x}|\theta)}{\partial \theta} &= n\theta^{n-1} e^{-\sum x_i \theta} + \theta^n e^{-\sum x_i \theta}(-\sum x_i) \\
0 &= n\theta^{n-1} e^{-\sum x_i \theta} + \theta^n e^{-\sum x_i \theta}(-\sum x_i) \\
0 &= n + \theta(-\sum x_i) \\
\hat{\theta} &= \frac{1}{\overline{x}}
\end{aligned}$$

or

$$\begin{aligned}
L(\theta) &= n \ln(\theta) - \sum x_i \theta \\
\frac{\partial L(\theta)}{\partial \theta} &= \frac{n}{\theta} - \sum x_i \\
0 &= \frac{n}{\theta} - \sum x_i \\
\hat{\theta} &= \frac{1}{\overline{x}} \\
\frac{\partial^2 L(\theta)}{\partial \theta^2} &= -\frac{n}{\theta^2} < 0 \\
&\text{maximum}
\end{aligned}$$