Math 152, Fall 2022
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WU # 21
Tuesday 12/1/22

Your name: ____

Names of people you worked with: ____

Task: Suppose a safety inspector needs to monitor the number of car accidents per month at a specific intersection. The inspector enters the number of monthly accidents in a worksheet where each value denotes the count of accidents in one month:

Accidents
2
0
2
2
4

To further model the data, the inspector wants to argue that the data have a Poisson distribution. Perform a goodness-of-fit test to evaluate whether the Poisson model can be rejected.

H_0 : data are distributed Poisson
H_1 : data are not distributed Poisson

You'll need to find the probability of seeing 0, 2 or 4 accidents if the Poisson model is correct. (Use $\hat{\lambda} = \overline{X}$ as the Poisson parameter.)

Solution:

Note: $\overline{x} = 2$.

value	count	Poisson probability
0	1	$P(X = 0) = e^{-2} = 0.135$
2	3	$P(X = 2) = e^{-2}2^2/2! = 0.271$
4	1	$P(X = 4) = e^{-2}2^4/4! = 0.09$

$$\begin{aligned} 2\ln(\Lambda(\underline{x})) &= 2\sum_{i=0}^{\infty} N_i \ln\left(\frac{N_i}{np_i^0}\right) \\ &= 2 \cdot \left(\ln\left(\frac{1}{5 \cdot 0.135}\right) + 3 \cdot \ln\left(\frac{3}{5 \cdot 0.271}\right) + \ln\left(\frac{1}{5 \cdot 0.09}\right)\right) \\ &= 7.15 \end{aligned}$$

Let's say that we were planning to group 5+ accidents (thus our data collection would be into $m = 6$ groups). The 0.95 quantile of a χ^2 distribution with df $= m - 1 - 1 = 4$ is `qchisq(0.95, 4) = 9.49`. So we do not reject the null hypothesis.

Note also that the p-value is $P(\chi^2_4 \geq 7.15) =$ `1 - pchisq(7.15, 4) = 0.128`.