Math 152, Fall 2022
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WU # 20
Tuesday 11/29/22

Your name: ____

Names of people you worked with: ____

Task: Consider a random sample $X_1, X_2, \ldots, X_n$ from Poisson(λ). The MLE is given by $\hat{\lambda} = \overline{X}$. We want to test the following hypotheses:
$H_0 : \lambda \leq 4$
$H_1 : \lambda > 4$

Let's say we know that $\overline{X} > 4$. What value of λ maximizes $f(\underline{x}|\lambda)$ when $\lambda \leq 4$?

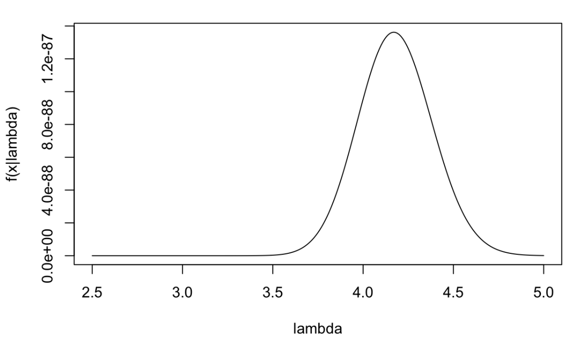
1. Calculate $f(\underline{x}|\lambda)$.
2. Sketch $f(\underline{x}|\lambda)$ (on the y-axis) as a function of λ (on the x-axis). [Feel free to use $n = 10$, $\overline{x} = 4.17$, and $\prod x_i! = 3 \times 10^{18}$ if you want to use desmos or Wolfram/Alpha.]
3. Proof by picture is fine. Or take derivatives.

Also, ask yourself, is $f(\underline{x}|\lambda)$ a discrete or continuous function when considering λ as the variable?

Solution:

$$f(\underline{x}|\lambda) = \frac{e^{-n\lambda}\lambda^{\sum_i x_i}}{\prod_i x_i!} \quad 0 \leq \lambda < \infty$$

Below is a plot from a sample of $n = 100$ points whose average is $\hat{\lambda} = \overline{x} = 4.17$. The maximum value of the function over $\lambda \leq 4$ happens at $\lambda = 4$.



How would you show it analytically? The first derivative shows that the function has a single optimum (at $\overline{X}$). A second derivative show you that the (single) optimum is a maximum. You can also see that the function is continuous in λ. If the function is continuous and has a max at a value which is larger than 4, we know that for all values less than 4, the maximum will happen when we get as close as possible to the global max (i.e., when $\lambda = 4$).