Math 152, Fall 2022
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WU # 1
Tueday 8/30/22

Your name / nickname / pronouns: ____

Names of people you worked with: ____

Task: Suppose the rate of infection with TB is 1 in 1000 (about 0.1 percent = 0.001). Suppose a TB test is used which is 90% accurate: it gives a positive result for 10 percent of people who do not actually have TB, but do have a reaction to the skin test. Also, 10% of the people who actually have TB fail to react to the test.

1. What's the chance that someone has TB if they test positive?
2. What's the chance that a randomly chosen person who tests negative actually has TB?
3. There is another TB test which gives fewer false positives, but it is more expensive. Would it be better to use that one?
4. (Hint: calculations are already performed.)
   - What is the **prior** probability of having TB?
   - What is the **posterior** probability of having TB (given a positive test)?

Solution:

We can use a table to figure out the probabilities. Consider a population with 10,000 people:

	Test +	Test -	Total
TB +	9	1	10
TB -	999	8,991	9,990
Total	1,008	8,992	10,000

Alternatively, we can use probability statements which will be easier to work with in the long-run as the scenarios get more complicated. We know the following:

$$\begin{aligned} P(TB+) &= 0.001 \\ P(Test+|TB-) &= 0.1 \\ P(Test-|TB+) &= 0.1 \end{aligned}$$

1. 10 in 10,000 people will have the disease. 9 of those 10 will actually test positive for TB. However, 999 of 9990 people will be false positives. so, only $9/(999+9) = 0.0089$ or $\approx 0.9\%$ of people who test positive actually have TB.

$$\begin{aligned} P(TB+|Test+) &= \frac{P(TB+\&Test+)}{P(Test+)} = \frac{P(Test+|TB+)P(TB+)}{P(Test+)} \\ &= \frac{P(Test+|TB+)P(TB+)}{P(Test+|TB-)P(TB-)+P(Test+|TB+)P(TB+)} \\ &= \frac{0.9\cdot 0.001}{0.1\cdot 0.999+0.9\cdot 0.001} \\ &= 0.0089 \end{aligned}$$

2. Of those who test negative, about 1 in 8,992 people will have a false negative. The total rate of false negatives in the population is 1 out of 10,000; the total rate of false positives in the population is 999 out of 10,000.

$$\begin{aligned}
P(TB+|Test-) &= \frac{P(TB+\&Test-)}{P(Test-)} = \frac{P(Test-|TB+)P(TB+)}{P(Test-)} \\
&= \frac{P(Test-|TB+)P(TB+)}{P(Test-|TB-)P(TB-)+P(Test-|TB+)P(TB+)} \\
&= \frac{0.1 \cdot 0.001}{0.9 \cdot 0.999 + 0.1 \cdot 0.001} \\
&= 0.00011121
\end{aligned}$$

3. Not necessarily, since it's much worse to have a false negative than a false positive. People who test positive are then given another test with fewer false positives.

4.
   - $P(TB+) = 0.001$
   - $P(TB+|Test+) = 0.009$