Math 152, Fall 2022
Jo Hardin
WU # 19
Tuesday 11/22/22

Your name: ______________________________

Names of people you worked with: ______________________________

Task:

Let $X_1, X_2, \ldots, X_n \sim Gamma(10, \theta), E[X] = 10/\theta$.
Find $T = r(\underline{X})$ such that for $\theta_1 < \theta_2$:

$$\frac{f(\underline{x}|\theta_2)}{f(\underline{x}|\theta_1)}$$

- depends on $\underline{x}$ only through T
- is a nondecreasing function of T over the range of possible values of T

Solution:

$$\begin{aligned}
f(x|\theta) &= \frac{\theta^{10}}{\Gamma(10)} x^{10-1} e^{-x\theta} \qquad 0 \leq x \leq \infty \qquad \text{let } \theta_1 < \theta_2 \\
\frac{f(\underline{x}|\theta_2)}{f(\underline{x}|\theta_1)} &= \frac{\theta_2^{10n}}{\theta_1^{10n}} e^{-\sum x_i \theta_2 + \sum x_i \theta_1} \\
&= \left(\frac{\theta_2}{\theta_1}\right)^{10n} e^{-\sum x_i(\theta_2 - \theta_1)} \\
T &= -\sum X_i
\end{aligned}$$

$f(\underline{X}|\theta)$ has a monotone likelihood ratio in $-\sum X_i$.