Math 152, Fall 2022
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WU # 18
Thursday 11/10/22

Your name: ______________________________________

Names of people you worked with: ______________________________________

Task: Let's say we have 2 batches of paint one of which is quick-dry. The paint is unlabeled, and we forgot which was which! We paint 5 boards from batch 1 and record the drying time. We think batch 1 is quick dry. We also believe that the drying times are normally distributed with a true st dev of $\sigma = 5$ min. The two batches of paint have a true average drying time of either 25 min or 10 min.

$$H_0 : \theta = 25 \text{ min}$$
$$H_1 : \theta = 10 \text{ min}$$

Using the 5 observed drying times, find δ^* that minimizes $\beta(\delta^*)$ subject to $\alpha(\delta^*) \leq .05$ (Note: the level of significance is set to be $\alpha_0 = 0.05$.)

Solution:

$$\delta^* : \{\text{reject } H_0 \text{ if } \overline{x} < 17.5 - \frac{5}{3n}\ln(k)\}$$

$$\begin{aligned}
P(\overline{X} < 17.5 - \frac{5}{3n}\ln(k) | \theta = 25) &= 0.05 \\
P(Z < \frac{17.5 - 5/3n\ln(k) - 25}{5/\sqrt{n}}) &= 0.05 \\
\frac{17.5 - 5/3n\ln(k) - 25}{5/\sqrt{n}} &= -1.645 \\
\ln(k) &= -11.47 \\
\delta^* : \{\text{reject } H_0 \text{ if } \overline{x} < 21.32\} & \\
\text{note: } P(\overline{X} > 21.32 | \theta = 10) &= 0
\end{aligned}$$

But wait, there is a really important idea here!!!

We could have done this problem without so much algebra. We know that the test must be based on $\overline{X}$ because that was the statistic which was isolated when the likelihood ratio was calculated.

$$\delta^* : \{\text{reject } H_0 \text{ if } \overline{x} < \text{ some constant }\}$$

So, we set the probability of rejecting H_0 when H_0 is true to 0.05.

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\begin{aligned}
\alpha(\delta^*) = P(\overline{X} < c|\theta = 25) &= 0.05 \\
P\left(Z < \frac{c - 25}{5/\sqrt{n}}\right) &= 0.05 \\
\frac{c - 25}{5/\sqrt{n}} &= -1.645 \\
c &= -1.645 * 5/\sqrt{(}5) + 25 \\
\delta^* : \{\text{reject } H_0 \text{ if } \overline{x} < 21.32\} &
\end{aligned}
$$

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\text{note: } \beta(\delta^*) = P(\overline{X} \geq 21.32|\theta = 10) \approx 0
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