Math 152, Fall 2022
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WU # 16
Thursday 11/3/22

Your name: ____

Names of people you worked with: ____

Task: Let's say we're going to flip a coin 100 times, and we assume the probability of heads is $\theta = 0.5$.

1. Use the normal approximation to the binomial to calculate the probability of getting 55 or more heads.
2. For what value of c is the probability of c or more heads no more than 0.05?

Recall that if $X \sim Bin(n, \theta)$ in large samples (big n), the normal distribution approximates the binomial distribution, $X \overset{\text{approx}}{\sim} N(n\theta, n\theta(1-\theta))$.

Solution:

1.

$$\begin{aligned}
\pi(\theta = 0.5|\delta) &= P(X > 55|\theta = 0.5) \\
&= P\left(Z > \frac{55 - n\theta}{\sqrt{n\theta(1-\theta)}}\right) \\
&= P\left(Z > \frac{55 - 50}{\sqrt{25}}\right) \\
&= P(Z > 1) = 1 - P(Z \leq 1) = 0.1587
\end{aligned}$$

2.

$$\begin{aligned}
\pi(\theta = 0.5|\delta) &= P(X > c|\theta = 0.5) \\
&\leq 0.05 \\
P(X < c|\theta = 0.5) &\geq 0.95 \\
P(Z < \frac{c - 50}{5}|\theta = 0.5) &\geq 0.95 \\
\frac{c - 50}{5} &\geq 1.645 \\
c &\geq 58.25
\end{aligned}$$

We let c be as small as possible, so c=58.25 (so you'd have to get at least 59 heads).