Math 152, Fall 2022
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WU # 14
Thursday 10/27/22

Your name: ____

Names of people you worked with: ____

Task: Suppose $X \sim$Poisson(θ). $f(x|\theta) = \frac{e^{-\theta}\theta^x}{x!}$. Find the Fisher Information in X (about θ) using all three methods (you should get three identical answers).

Solution:

$$\begin{aligned}
\lambda(x|\theta) &= \ln f(x|\theta) = -\theta + x \ln \theta - \ln(x!) \\
\lambda'(x|\theta) &= -1 + x/\theta \\
\lambda''(x|\theta) &= -x/\theta^2
\end{aligned}$$

$$\begin{aligned}
I(\theta) &= -E[\lambda''(X|\theta)] = -E[-X/\theta^2] \\
&= \theta/\theta^2 = 1/\theta
\end{aligned}$$

$$\begin{aligned}
I(\theta) &= var(\lambda'(X|\theta)) = var(-1 + X/\theta) \\
&= \frac{1}{\theta^2} var(X) = \theta/\theta^2 = 1/\theta
\end{aligned}$$

$$\begin{aligned}
I(\theta) &= E\{[\lambda'(X|\theta)]^2\} \\
&= E[1 - 2X/\theta + X^2/\theta^2 \\
&= 1 - \frac{2}{\theta}E[X] + \frac{1}{\theta^2}E(X^2) \\
&= 1 - \frac{2}{\theta}\theta + \frac{1}{\theta^2}(var(X) + E[X]^2) \\
&= 1 - 2 + \frac{1}{\theta^2}(\theta + \theta^2) = 1/\theta
\end{aligned}$$

Notice that the information about θ in X depends on θ. This isn't always true.