Math 152, Fall 2022
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WU # 11
Thursday 10/6/22

Your name: ______________________________

Names of people you worked with: ______________________________

Task:

Consider the result derived in class:

$$\left(\frac{\lambda_1 \alpha_1}{\beta_1}\right)^{1/2} (\mu - \mu_1) \mid \underline{x} \sim t_{2\alpha_1} \tag{1}$$

where $\mu_1 = \frac{\lambda_0 \mu_0 + n\overline{x}}{\lambda_0 + n}$, $\lambda_1 = \lambda_0 + n$, $\alpha_1 = \alpha_0 + \frac{n}{2}$, $\beta_1 = \beta_0 + \frac{1}{2}\sum_{i=1}^n (x_i - \overline{x})^2 + \frac{n\lambda_0(\overline{x} - \mu_0)^2}{2(\lambda_0 + n)}$.

Provide the formula for a 98% posterior interval for μ, in terms of $\mu_1, \lambda_1, \alpha_1, \beta_1$.

Solution:

Let the confidence level be $1 - \alpha$. As with frequentist CI, the interval can be built by pivoting around the value of interest, μ.

$$\begin{aligned}
P(-c \leq U \leq c \mid \underline{x}) &= 1 - \alpha \\
P\left(-c \leq \left(\frac{\lambda_1 \alpha_1}{\beta_1}\right)^{1/2} (\mu - \mu_1) \leq c \mid \underline{x}\right) &= 1 - \alpha \\
P\left(\mu_1 - c\left(\frac{\beta_1}{\lambda_1 \alpha_1}\right)^{1/2} \leq \mu \leq \mu_1 + c\left(\frac{\beta_1}{\lambda_1 \alpha_1}\right)^{1/2} \mid \underline{x}\right) &= 1 - \alpha
\end{aligned}$$

$\Rightarrow \quad \mu_1 \pm c\left(\frac{\beta_1}{\lambda_1 \alpha_1}\right)^{1/2}$ is a $(1-\alpha)100\%$ posterior interval for μ.

And $c =$ `qt(1 - ` α`/2, 2`α_1`)`, here $c =$ `qt(0.99,`α_1`)`.