Math 152, Fall 2022
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WU # 10
Tuesday 10/4/22

Your name: ____

Names of people you worked with: ____

Task:

According to some investors, foreign stocks have the potential for high yield, but the variability in their dividends may be great. Let's say we take a random sample of 10 foreign stocks and assume that they come from a normal distribution. The data produce a sample mean of \$1 dividend per share with a sample standard deviation of \$2.

Find a 95% CI for the true SD (i.e., σ) associated with dividends on foreign stocks.

Hint: you will need to use the χ^2 distribution and one of the statistics discussed previously in class to start the problem. Try writing an equation of the following form:

$$P(\text{a number} \leq \text{statistic and the parameter of interest} \leq \text{another number}) = 0.95$$

Solution:

If the dividends are actually normally distributed (not really a reasonable assumption here), then theory tells us that:

$$\frac{\sum(X_i - \overline{X})^2}{\sigma^2} \sim \chi^2_9$$

Using σ to pivot (that is, putting σ in the middle of the probability statement) we get:

$$\begin{aligned}
P\left(\sum(X_i - \overline{X})^2/\chi^2_{9,.975} \le \sigma^2 \le \sum(X_i - \overline{X})^2/\chi^2_{9,.025}\right) &= 0.95 \\
P\left(\sqrt{\sum(X_i - \overline{X})^2/\chi^2_{9,.975}} \le \sigma \le \sqrt{\sum(X_i - \overline{X})^2/\chi^2_{9,.025}}\right) &= 0.95 \\
P\left(\sqrt{s^2 \cdot (n-1)/\chi^2_{9,.975}} \le \sigma \le \sqrt{s^2 \cdot (n-1)/\chi^2_{9,.025}}\right) &= 0.95
\end{aligned}$$

[Important!!!] where X_i (or s^2) is the random variable, the only thing which varies in the above probability statement.Otherwise, it doesn't make any sense to be talking about a probability, you are a **frequentist** today, after all.

Therefore, the 95% confidence interval for σ is: $\left(\sqrt{s^2 \cdot (n-1)/\chi^2_{9,.975}}, \sqrt{s^2 \cdot (n-1)/\chi^2_{9,.025}}\right)$.

Note that $\chi^2_{9,.025} = 2.7$ and $\chi^2_{9,.975} = 19.02$. Giving a 95% CI for σ of:

$$\left(\sqrt{2^2 \cdot 9/19.02}, \sqrt{2^2 \cdot 9/2.7}\right)$$
$$(1.38, 3.65)$$

The correct interpretation is: we are 95% confident that the true standard deviation of dividends on foreign stocks in the population from which these data were randomly sampled is between \$1.38 and \$3.65. [Fun fact: unlike the t-CI for the mean, the method applied above is very sensitive to the normality assumption.]